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Seiberg-Witten maps from the point of view of consistent deformations of gauge theories

Glenn Barnich* **and Marc Henneaux**†

Physique Théorique et Mathématique, Université Libre de Bruxelles
Campus Plaine C.P. 231, B–1050 Bruxelles, Belgium
E-mail: gbarnich@ulb.ac.be, henneaux@ulb.ac.be

Maxim Grigoriev

Lebedev Physics Institute, Leninisky Prospect 53
Moscow 117924, Russia
E-mail: grig@lpi.ru

Abstract: Non-commutative versions of theories with a gauge freedom define (when they exist) consistent deformations of their commutative counterparts. General aspects of Seiberg-Witten maps are discussed from this point of view. In particular, the existence of the Seiberg-Witten maps for various non-commutative theories is related to known cohomological theorems on the rigidity of the gauge symmetries of the commutative versions. In technical terms, the Seiberg-Witten maps define canonical transformations in the antibracket that make the solutions of the master equation for the commutative and non-commutative versions coincide in their antifield-dependent terms. As an illustration, the on-shell reducible non-commutative Freedman-Townsend theory is considered.

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*Research Associate of the Belgium National Fund for Scientific Research.
†Also at Centro de Estudios Cientifícos, Casilla 1469, Valdivia, Chile.

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1. Introduction

A remarkable feature of Yang-Mills theory is the (formal) rigidity of its gauge structure. Namely, there is no consistent deformation of the Yang-Mills action

$$I_{YM}[A_\mu] = -\frac{1}{4}\int d^n x\ F^a_{\mu\nu}F^{\mu\nu}_a \tag{1.1}$$

that truly deforms its gauge symmetries. By allowed redefinitions of the gauge parameters and of the fields, one can always bring the gauge transformations of the deformed theory back to the original form

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a + f^a{}_{bc} A^b_\mu \epsilon^c \,. \tag{1.2}$$

In this light, the existence of the so-called Seiberg-Witten (SW) map [1] for the non-commutative deformation of (1.1) has a clear underlying algebraic rationale. [For a recent review on non-commutative Yang-Mills theories, see [2].]

The rigidity of the gauge symmetries of (1.1) was established in [3, 4] by cohomological techniques, without assuming Lorentz invariance or restricting the class of possible deformations to those with pre-assigned "engineering dimension". This is particularly relevant to the case where a prescribed ("external") constant matrix $\theta^{\mu\nu}$ with (negative) mass dimension is present.[1]

In this paper, we discuss Seiberg-Witten maps from the point of view of consistent deformations of gauge theories in the context of the Batalin-Vilkovisky formalism for local gauge theories [6]–[15] (for reviews, see e.g. [16, 17]). In particular, we derive cohomological conditions that guarantee the existence of Seiberg-Witten maps. The SW maps turn out to be in fact canonical transformations in the antibracket that generically mix the original fields, the ghosts and the antifields. The antifields do not occur in the transformation of the fields and ghosts for the Yang-Mills case because the gauge structure is defined then not just on-shell but also off-shell. However, they do occur for more general gauge theories.

We illustrate this feature for the non-commutative deformation of the Freedman-Townsend [18], whose gauge structure is rigid (see [19] and section 3.3). This deformation exhibits clearly the new features of generic gauge theories which are: (i) the gauge symmetries are reducible, so ghosts of ghosts are necessary and must be considered in the SW canonical transformations (corresponding to possible redefinitions of the reducibility coefficients); (ii) reducibility holds only on-shell and so, one cannot separate the symmetries from the dynamics, as one can in the Yang-Mills case. We explicitly construct the generating functional of the SW map in the u(1)-case and point out the mixture of the antifields with the fields, which precisely reflects the mixing of the dynamics with the symmetries.

Our conclusions are presented in section 5.

Finally, we mention a few references where other approaches to SW maps in the Yang-Mills case are discussed. Existence by explicit construction has been shown in e.g. [20], where commutative and non-commutative versions of Wilson lines were compared and in [21], where the SW map was computed in the framework of Kontsevich's

[1] Actually, rigidity holds only if the gauge group has no abelian factor. When there is an abelian factor, there can be deformations of (1.1) that truly deform the gauge structure; these involve conserved currents [3]. One can easily show that such a possibility does not arise in the non-commutative deformation of U(N) Yang-Mills theory because of the derivative structure of the coupling, see [5].

approach to deformation quantization. A reference that also uses cohomological arguments (though not in the BV formalism) is [22]. An explicit inverse SW map was given in [23].

2. Seiberg-Witten maps: general considerations

2.1 Non-commutative gauge theories as consistent deformations of commutative gauge theories

Consider a gauge theory with action $I_0[\hat{\varphi}] = \int d^n x \; L_0([\hat{\varphi}])$, where the notation $f = f([y])$ means that the function f depends on the variable y and a finite number of its derivatives. We put a hat on the fields in anticipation of changes of field variables performed below to unhatted fields. We denote the infinitesimal gauge transformations by $\delta_{0,\epsilon}\hat{\varphi}^i = R^i_0([\hat{\varphi}],[\hat{\epsilon}])$ (the dependence on the gauge parameters $\hat{\epsilon}^\alpha$ and their derivatives is of course linear). One form of the Noether identities expressing gauge invariance is

$$\frac{\delta L_0}{\delta \hat{\varphi}^i} R^i_0 + \partial_\mu j^\mu = 0\,, \tag{2.1}$$

for some j^μ, where $\delta L_0/\delta\hat{\varphi}^i$ are the Euler-Lagrange derivatives of L_0. The gauge theory may be reducible, i.e. there may exist particular choices of the gauge parameters, $\hat{\epsilon}^\alpha = {Z_0}^\alpha([\varphi],[\hat{\eta}])$ for which the gauge transformations are trivial,

$$R^i_0([\varphi],[Z_0]) \approx 0\,, \tag{2.2}$$

where $\approx$ means on-shell for the equations of motion defined by I_0. As we have indicated, there may be more than one reducibility condition, i.e. the ${Z_0}^\alpha$'s may depend (linearly) on a certain number of reducibility parameters $\hat{\eta}^A$ and their derivatives.

A (formal) consistent deformation of such a gauge theory is defined by giving (i) a deformed action, (ii) deformed gauge symmetries and (iii) deformed reducibility expressions

$$\begin{aligned}
\hat{I} &= I_0 + gI_1 + \frac{1}{2}g^2 I_2 + \cdots\,,\\
\hat{R}^i &= R^i_0 + gR^i_1 + \frac{1}{2}g^2 R^i_2 + \cdots\,,\\
\hat{Z}^\alpha &= {Z_0}^\alpha + g{Z_1}^\alpha + \frac{1}{2}g^2 {Z_2}^\alpha + \cdots\,,
\end{aligned} \tag{2.3}$$

such that the Noether identities and the reducibility equations are preserved order by order in the deformation parameter g,

$$\begin{aligned}
\frac{\delta \hat{L}}{\delta\hat{\varphi}^i}\hat{R}^i + \partial_\mu \hat{j}^\mu \;&=\; 0\,,\\
\hat{R}^i([\varphi],[\hat{Z}]) \;&\approx'\; 0\,,
\end{aligned} \tag{2.4}$$

where $\approx' 0$ means on-shell for the equations of motions defined by the complete action $\hat{I} = \int d^n x \, \hat{L}$. The deformed lagrangian $\hat{L}$ may involve all the derivatives of the fields, but we require that each term L_i in the power expansion $\hat{L} = L_0 + gL_1 + \cdots$ be a local function, i.e. contains a finite number of derivatives.

With these definitions, non-commutative extensions of commutative gauge theories are clearly (when they exist) consistent deformations of their commutative counterparts, the deformation parameter g multiplying the matrix $\theta^{\mu\nu}$ defining the non-commutativity of the coordinates (see below).

2.2 Gauge structure

As explained e.g. in [16], the most general gauge transformation is obtained by adding to the transformations $\delta_{0,\epsilon}\hat{\varphi}^i = R_0^i$ (or $\hat{R}^i$), in which the gauge parameters are chosen arbitrarily (in particular, can be functions of the fields and their derivatives), an arbitrary antisymmetric combination of the equations of motion. Because the most general gauge transformation explicitly refers to the dynamics, the algebra of all the gauge transformations of pure Maxwell theory, say, is different from the algebra of Born-Infeld theory. However, in both cases, the relevant information about the gauge transformations is contained in the transformation $\delta A_\mu = \partial_\mu \epsilon$, which is identical for the two theories. Only the "on-shell trivial" gauge transformations, involving the equations of motion, are different. For this reason, one says that pure Maxwell theory and Born-Infeld theory have identical gauge structures.

More generally, one says that two gauge theories have identical gauge structures if it is possible to choose their generating sets such that they coincide. The generating sets are precisely the subsets of the gauge algebra that contain the relevant information about all the gauge symmetries, in the sense that any gauge transformation can be obtained from the generating set by choosing appropriately the gauge parameters (possibly, as functions of the fields) and adding if necessary an on-shell trivial gauge symmetry [16]. There is a huge freedom in the choice of generating sets. Furthermore, one may also redefine the reducibility relations.[2] Moreover, the equivalence may become manifest only after one has redefined the field variables. So in order to show that two theories have identical gauge structures, one must establish the existence of a field transformation such that the two generating sets can be made to coincide (through redefinitions of the gauge parameters and addition of on-shell trivial gauge transformations).

One virtue of the antifield formalism is that all this freedom can be neatly taken into account trough canonical transformations (see e.g. [12, 16, 24]). For this reason, we review how consistent deformations are formulated in the antifield (BV) formalism.

[2] Note that if reducibility holds only on-shell, the equations of motion that appear in the reducibility identities must be the same in both theories — when written in the same variables.

2.3 Cohomological reformulation of consistent deformations in the BV formalism

In the framework of the BV formalism, all the information on the invariance of the action and the algebra of gauge symmetries is encoded in an extended action satisfying the so-called master equation. The problem of consistent deformations of gauge theories can then be reformulated [25] (see [26] for a review) as the problem of deforming the solution of the master equation in the space of local functionals (while maintaining the master equation itself). In the present context, local functions are formal power series in the deformation parameter, each term depending on the original fields, the ghosts, ghosts for ghosts, their antifields and a finite number of derivatives of all these fields. Local functionals are identified with equivalence classes of local functions up to total divergences (see [27] for more details). We shall denote the solution of the (classical) master equation for the original and deformed theories by S_0 and $\hat{S}$, respectively. One has

$$\begin{aligned} S_0 &= I_0 + \int d^n x\ \hat{\varphi}^*_i R^i_0\left([\hat{\varphi}],\left[\hat{C}\right]\right) + \hat{C}^*_\alpha {Z_0}^\alpha([\hat{\varphi}],[\hat{\rho}]) + \cdots\,, \\ \frac{1}{2}(S_0,S_0) &= 0\,, \\ \hat{S} &= S_0 + gS_1 + \left(\frac{1}{2}\right) g^2 S_2 + \cdots \\ &= \hat{I} + \int d^n x\ \hat{\varphi}^*_i \hat{R}^i\left([\hat{\varphi}],\left[\hat{C}\right]\right) + \hat{C}^*_\alpha \hat{Z}^\alpha([\hat{\varphi}],[\hat{\rho}]) + \cdots\,, \\ \frac{1}{2}(\hat{S},\hat{S}) &= 0\,. \end{aligned} \tag{2.5}$$

Here, the $\hat{C}^\alpha$ are the ghosts replacing the gauge parameters, the $\hat{\rho}^A$ are the ghosts for ghosts, while $\hat{\varphi}^*_i$, $\hat{C}^*_\alpha$ and $\hat{\rho}^*_A$ are the associated antifields, of respective antifield number $1,2,3$ (see e.g. [9, 16, 17] for details).

Note that the antifield-independent part of the solution of the master equation is just the classical action. The information about the gauge symmetries, their algebra, the reducibility equations etc. is contained in the antifield-dependent part. *Thus, if the original and deformed theories have the same gauge structure, S_0 and $\hat{S}$ have the same antifield-dependent part and vice-versa.* The advantage of the cohomological reformulation is that standard techniques of deformation theory can now be applied.

In particular, (2.5) implies that

$$\left(\hat{S},\frac{\partial \hat{S}}{\partial g}\right) = 0\,, \tag{2.6}$$

which in turn implies that an infinitesimal deformation $\frac{\partial \hat{S}}{\partial g}|_{g=0} = S_1$ is a cocycle of the BRST differential $s_0 = (S_0,\cdot)$ of the undeformed theory,

$$(S_0,S_1) = 0\,. \tag{2.7}$$

A deformation is trivial if it can be undone through a canonical, i.e. antibracket preserving, transformation:[3]

$$\hat{S}\left[\hat{\phi}([\phi],[\phi^*];g),\hat{\phi}^*([\phi],[\phi^*];g);g\right] = S_0[\phi,\phi^*]\,, \tag{2.8}$$

Here — and throughout below —, the original fields and ghosts are collectively denoted by $\hat{\phi}$, while $\hat{\phi}^*$ denotes collectively all the antifields. Thus, a generic canonical transformation mixes the original fields, the ghosts and the antifields. Equivalently, this means that $\hat{S}$ can be obtained from the original S_0 through the inverse canonical transformation

$$\hat{S}\left[\hat{\phi},\hat{\phi}^*;g\right] = S_0\left[\phi\left(\left[\hat{\phi}\right],\left[\hat{\phi}^*\right];g\right),\phi^*\left(\left[\hat{\phi}\right],\left[\hat{\phi}^*\right];g\right)\right]. \tag{2.9}$$

This is the case iff the cocycle $\partial\hat{S}/\partial g$ is a coboundary of the deformed theory,

$$\frac{\partial\hat{S}}{\partial g} = \left(\hat{S},\hat{\Xi}\right). \tag{2.10}$$

Indeed, if (2.10) holds, then a canonical transformation with the required properties is given by

$$\begin{aligned}\phi^A(x) &= P\exp\left(\int_0^g dg'\left(\cdot,\hat{\Xi}(g')\right)\right)\hat{\phi}^A(x)\,,\\ \phi^*_A(x) &= P\exp\left(\int_0^g dg'\left(\cdot,\hat{\Xi}(g')\right)\right)\hat{\phi}^A(x)\,.\end{aligned} \tag{2.11}$$

Conversely, if the deformed action can be obtained from the undeformed one by a canonical transformation for any g, then the passage from $\hat{S}(g)$ to $\hat{S}(g+\delta g)$ is an infinitesimal canonical transformation (by the group property of canonical transformations) and (2.10) holds. It will be useful in the sequel to introduce the generating functional $F[\phi,\hat{\phi}^*;g]$ of "second type" in ghost number -1 such that

$$\hat{\phi}^A(x) = \frac{\delta^L F}{\delta\hat{\phi}^*_A(x)}\,,\qquad \phi^*_A(x) = \frac{\delta^L F}{\delta\phi^A(x)}\,, \tag{2.12}$$

(see [12] and [28, appendix A] for material on antibracket preserving transformations).

It follows from (2.6) and (2.10) that a necessary condition for the existence of a non-trivial deformation is the existence of a non-trivial cohomology class for the deformed theory. Because every cocycle of the BRST differential $\hat{s} = (\hat{S},\cdot)$ of the deformed theory gives to lowest order in g a cocycle of the BRST differential $s_0 =$

[3]Only canonical transformations that reduce to the identity to order 0 in the deformation parameter are considered here. Invertibility of these transformations in the space of formal power series is then guaranteed.

$(S_0, \cdot)$ of the undeformed one, and furthermore, every coboundary of the undeformed theory can be extended to a coboundary of the deformed theory, it follows that non-trivial deformations are controlled by the local BRST cohomology of the undeformed theory. In particular, if this cohomology is empty in the relevant subspace of the space of local functionals, it follows that the deformation is trivial. By relevant subspace, we mean the subspace of local functionals of ghost number 0 possibly restricted through additional requirements like global symmetries or power counting restrictions, depending on the problem at hand.

Elements of $H^0(s_0)$ are called non-trivial infinitesimal deformations. One can furthermore show that the obstruction to extending infinitesimal deformations are controlled by the antibracket map

$$(\cdot,\cdot)_M : H^0\left(s^{(0)}\right) \otimes H^0\left(s^{(0)}\right) \longrightarrow H^1\left(s^{(0)}\right) \tag{2.13}$$

but this will not be needed here.

2.4 SW maps in the context of deformation theory

Given a non-trivial consistent deformation of a gauge theory, one may ask whether the gauge symmetries, their algebra and their reducibilities are equivalent to the one of the undeformed theory through allowed redefinitions of the most general type. The allowed redefinitions of the gauge symmetries can involve redefinitions of the gauge parameters that contain the fields themselves, as well as the possible addition of on-shell trivial symmetries [16]. Note that if the classical action $\hat{I}$ is equivalent through field redefinitions to the action I_0, then the gauge symmetries and their structure are of course equivalent, whereas the converse does not hold (e.g. as explained above, Maxwell theory and Born-Infeld theory are based on inequivalent actions but their gauge structures are identical).

Because the gauge symmetries and their structure are described in the master equation through terms with strictly positive antifield number, this question amounts to the question of the existence of a canonical transformation $\hat{\phi}[\phi,\phi^*;g], \hat{\phi}^*[\phi,\phi^*;g]$ such that

$$\hat{S}\left[\hat{\phi}[\phi,\phi^*;g],\hat{\phi}^*[\phi,\phi^*;g];g\right] = S_0[\phi,\phi^*] + V[\varphi;g]\,. \tag{2.14}$$

That is, if one can absorb all the antifield-dependence through a canonical transformation, the only effect of the deformation will be indeed (after redefinitions) just to change the original action $I_0[\varphi]$ into $I^{\text{eff}}[\varphi;g] \equiv I_0[\varphi] + V[\varphi;g]$,

$$\begin{aligned} I_0[\varphi] \longrightarrow I^{\text{eff}}[\varphi;g] &= I_0[\varphi] + V[\varphi;g]\,, \\ V[\varphi;g] &= gV_1[\varphi] + \frac{g^2}{2}V_2[\varphi] + \cdots \end{aligned} \tag{2.15}$$

without modifying the gauge transformations. The fact that one considers general canonical transformations automatically incorporates all the available freedom (see e.g. [16]). In particular, it allows for redefinitions of the ghosts that involve the original fields, or, what is the same in terms of the gauge transformations, redefinitions of the gauge parameters that contain the fields φ^i. It also allows for redefinitions of the gauge transformations that involve on-shell trivial gauge symmetries. In short, it enables one to go from one generating set to any other generating set.

By a reasoning similar to that of the previous paragraph, the condition (2.14) can be shown to be equivalent to the condition

$$\frac{\partial \hat{S}}{\partial g} = \frac{\partial V}{\partial g} + \left(\hat{S}, \hat{\Xi}\right). \tag{2.16}$$

Note also that eq. (2.14) reads, in the variables $\phi, \hat{\phi}^*$,

$$\hat{S}\left[\frac{\delta^L F}{\delta \hat{\phi}^*}, \hat{\phi}^*; g\right] = S_0\left[\phi, \frac{\delta^L F}{\delta \phi}\right] + V[\varphi; g]. \tag{2.17}$$

The canonical transformations that achieve (2.14) or (2.17) will be called Seiberg-Witten (SW) maps.

2.5 Existence of Seiberg-Witten maps

An important instance where (2.16) holds arises in the following situation. To lowest order in g, condition (2.16) reduces to

$$S_1 = V_1(\hat{\varphi}) + (S_0, \Xi_1), \tag{2.18}$$

for the s_0 cocycle $\hat{S}_1$, with $\Xi_1 = \hat{\Xi}[\hat{\phi}, \hat{\phi}^*, 0]$. Suppose that in the relevant subspace in which the deformation is allowed to take place, the representatives of all the cohomology of s_0 can be chosen to be antifield independent, i.e.

$$(S_0, C) = 0 \Longrightarrow C = C'[\varphi] + (S_0, D) \tag{2.19}$$

for some $C'[\varphi]$ that depends only on the original fields and not on the antifields (and which is of course annihilated by s_0, $s_0 C' = 0$). Then (2.14) can be achieved through a succession of canonical transformations. Indeed, let $\hat{z} = (\hat{\phi}, \hat{\phi}^*)$ and consider the canonical transformation $z^1 = \exp(g(\cdot, \Xi^{(1)}))\hat{z}$, so that $\hat{z} = z^1 - g(z^1, \Xi^{(1)}) + O(g^2)$. It follows that

$$\hat{S}\left[\hat{z}\left(\left[z^1\right]; g\right); g\right] = S_0\left[z^1\right] + gV_1\left[\varphi^1\right] + g^2 \tilde{S}^{(2)}\left[z^1\right] + O\left(g^3\right), \tag{2.20}$$

for some $\tilde{S}^{(2)}[z^1]$ and $s_0 V_1 = 0$. More generally, assume that the Seiberg-Witten map has been constructed to order k, i.e. that we have constructed a canonical transformation $z^k = z^k[\hat{z}; g]$ such that

$$\hat{S}\left[\hat{z}\left[z^k; g\right]; g\right] = S_0\left[z^k\right] + \sum_{i=1}^{k} g^i V_i\left[\varphi^k\right] + g^{k+1}\tilde{S}^{(k+1)}\left[z^k\right] + O\left(g^{k+2}\right), \tag{2.21}$$

with $s_0 V_i = 0$. The equation $\frac{1}{2}(\hat{S}[\hat{z}[z^k;g]], \hat{S}[\hat{z}[z^k;g]]) = 0$, which holds because the transformation is canonical then gives to lowest non-trivial order g^{k+1} that $(S_0[z^k], \tilde{S}^{(k+1)}[z^k]) = 0$, which in turn implies by the assumption (2.19) that

$$\tilde{S}^{(k+1)}\left[z^k\right] = V_{k+1}\left[\varphi^k\right] + \left(S_0\left[z^k\right], \Xi_{k+1}\left[z^k\right]\right). \tag{2.22}$$

The next canonical transformation is then $z^{k+1} = \exp(g^{k+1}(\cdot, \Xi_{k+1}[z^k]))\hat{z}^k$ so that $z^k = z^{k+1} - g^{k+1}(z^{k+1}, \Xi_{k+1}) + O(g^{k+2})$. If $\hat{z}[z^{k+1};g] = \hat{z}[z^k[z^{k+1};g];g]$, we have

$$\hat{S}\left[\hat{z}\left[z^{k+1};g\right];g\right] = S_0\left[z^{k+1}\right] + \sum_{i=1}^{k+1} g^i V_i\left[\varphi^{k+1}\right] + g^{k+2}\tilde{S}^{(k+2)}\left[z^{k+1}\right] + O\left(g^{k+3}\right), \tag{2.23}$$

for some $\tilde{S}^{(k+2)}[z^{k+1}]$ with $s_0 V_i = 0$, which proves that if (2.19) holds, the full Seiberg-Witten map can be obtained through an iteration of canonical transformations:

$$z = \Pi_{k=1}^{\infty} \exp(g^k(\cdot, \Xi_k))\hat{z}\,. \tag{2.24}$$

This discussion is in fact identical to the analysis of the renormalization of non-power counting renormalizable gauge theories as discussed in [29, 30].

Now, the crucial cohomological property (2.19) that implies the existence of the Seiberg-Witten map holds in the Yang-Mills case. This is the content of the cohomological theorems of [3, 4] proved in the context of general deformations not limited by power-counting restrictions or Lorentz-invariance. [As stated above, there is an antifield dependent deformation in the presence of a U(1) factor (see also [31]), as for U(N) gauge groups; however, it is easy to see that the non-commutative deformation of Yang-Mills theory has no component along this deformation because of its derivative structure, see [5].] The fact that the Seiberg-Witten map for non-commutative U(1) Yang-Mills theory can be extended to a canonical transformation in field-antifield space follows from the general properties relating canonical transformations to redefinitions of the fields and of the gauge transformations (see e.g. [16]) and has been explicitly verified in [32] (see also [33]). The cohomological property (2.19) also holds for the Chern-Simons theory — where in fact one has the stronger result that V_i is proportional to the original action [27] —, providing a cohomological understanding of the results of [34]. In all these cases, there is even a constructive (though somewhat cumbersome) procedure for explicitly removing the antifield-dependent terms and finding the generators of the successive canonical transformations that bring the gauge structure back to its original form through the use of homotopy formulas. A similar situation prevails for the non-commutative Freedman-Townsend model, as we shall analyse in section 4.

Finally, it is also possible to analyze along the above lines a situation where for instance the deformation of the action and the gauge symmetries are non trivial, while the algebra and higher order structure constants of the gauge symmetries are equivalent.

2.6 Weak Seiberg-Witten gauge equivalence

By expanding the condition (2.14) for the existence of the Seiberg-Witten maps according to the antifield number, one recovers formulas familiar from the Yang-Mills context (but modified to weak relations).

For instance, if we denote by $f^i([\varphi];g)$ and $g^\alpha([\varphi],[C];g)$ the expression of the hatted fields and ghosts in terms of the original fields and ghosts when the antifields are set equal to zero in (2.12),

$$f^i([\varphi];g) = \frac{\delta^L F}{\delta \hat{\varphi}^*_i}\bigg|_{\phi^*=0}, \qquad g^\alpha([\varphi],[C];g) = \frac{\delta^L F}{\delta \hat{C}^*_\alpha}\bigg|_{\phi^*=0} \tag{2.25}$$

the condition (2.14) becomes at antifield number zero

$$\hat{I}[f;g] = I^{\text{eff}}[\varphi;g]\,. \tag{2.26}$$

In antifield number 1, one gets

$$\hat{\delta}_{\hat{\lambda}}\hat{\varphi}^i \approx \delta_\lambda \hat{\varphi}^i\,, \tag{2.27}$$

where the even gauge parameter $\hat{\lambda}$ corresponds to the odd function $g^\alpha([\varphi],[C],g)$ while λ corresponds to C^α. The relation (2.27) generalizes, in the open, reducible algebra case, the key relation of [1] that defines the Seiberg-Witten maps.

Finally, in antifield number 2, one gets an integrability condition for (2.27), as well as possible (admissible) redefinition of the reducibility functions. The integrability condition is the BRST version of the Wess-Zumino type consistency condition [35] that one can deduce directly from the weak Seiberg-Witten equivalence condition (2.27). In higher antifield number, one gets higher-order integrability conditions related to the existence of higher-order structure functions.

3. Non-commutative Freedman-Townsend model

3.1 Preliminaries

We assume from now on the space-time manifold to be $\mathbb{R}^n$ with coordinates x^μ, $\mu = 1,\ldots,n$. The Weyl-Moyal star-product is defined through

$$f * g(x) = \exp(i\wedge_{12}) f(x_1) g(x_2)|_{x_1=x_2=x}\,, \qquad \wedge_{12} = \frac{g}{2}\theta^{\mu\nu}\partial^{x_1}_\mu \partial^{x_2}_\nu\,, \tag{3.1}$$

for a real, constant, antisymmetric matrix $\theta^{\mu\nu}$. The parameter g is the deformation parameter. Standard formulas are recovered by taking $g = 1$, while the commutative case corresponds to $g = 0$.

Let $M = m^A(x)T_A$ with T_A generators of the Lie algebra $\mathrm{u}(N)$, i.e. antihermitean $N\times N$ matrices. The coefficients $m^A(x)$ are real, commuting or anticommuting fields. If hermitean conjugation for the multiplication of $\mathbb{Z}_2$ graded functions is defined by

$(mn)^\dagger = (-)^{|m||n|} nm$, then hermitean conjugation of matrix valued function also satisfies $(MN)^\dagger = (-)^{|m||n|} NM$. We denote the invariant metric $\mathrm{Tr}\ T_A T_B$ by g_{AB}, $\mathrm{Tr}\ T_A T_B = g_{AB}$. It is non degenerate.

The graded Moyal bracket defined by

$$[M \overset{*}{,} N] = M * N - (-)^{|M||N|} N * M \tag{3.2}$$

is again a $\mathrm{u}(N)$ valued function, because $(M * N)^\dagger = (-)^{|M||N|} N * M$. This is a straightforward extension of the reasoning of [36] in the case where one allows the functions to belong to a $\mathbb{Z}_2$ graded algebra. Furthermore, the covariant derivative and associated field strength are defined as follows:

$$\begin{aligned} \hat{D}_\mu M &= \partial_\mu M + \left[\hat{A}_\mu \overset{*}{,} M\right], \\ \left[\hat{D}_\mu, \hat{D}_\nu\right] M &= \left[\hat{F}_{\mu\nu} \overset{*}{,} M\right], \\ \hat{F}_{\mu\nu} &= \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + \left[\hat{A}_\mu \overset{*}{,} \hat{A}_\nu\right]. \end{aligned} \tag{3.3}$$

A key property of the Moyal star-product is

$$M * N = MN + \partial_\mu \Lambda^\mu . \tag{3.4}$$

As a consequence, if boundary terms are neglected,

$$\begin{aligned} \int d^n x\ \mathrm{Tr}\ M * N &= \int d^n x\ \mathrm{Tr}\ MN = \int d^n x\ \mathrm{Tr}\ N * M (-)^{|M||N|}, \\ \int d^n x\ \mathrm{Tr}\ M * N * O &= \int d^n x\ \mathrm{Tr}\ O * M * N (-)^{|O|(|M|+|N|)}, \\ \int d^n x\ \mathrm{Tr}\ M * [N \overset{*}{,} O] &= \int d^n x\ \mathrm{Tr}\ [M \overset{*}{,} N] * O, \\ \int d^n x\ \mathrm{Tr}\ \hat{D}_\mu M * N &= - \int d^n x\ \mathrm{Tr}\ M * \hat{D}_\mu N . \end{aligned} \tag{3.5}$$

3.2 Action and gauge algebra of non-commutative FT model

The non-commutative $\mathrm{U}(N)$ Freedman-Townsend model exists in four dimensions. It is most conveniently formulated in first order form. The action is

$$\hat{I} = \int d^4 x\ \mathrm{Tr}\ \left(-\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \hat{B}_{\mu\nu} * \hat{F}_{\rho\sigma} + \frac{1}{2} \hat{A}_\mu * \hat{A}^\mu \right) \tag{3.6}$$

in Minkowski space-time, with signature $(- + ++)$, $\epsilon^{0123} = 1$ and $\epsilon_{0123} = -1$. The action is invariant under the gauge transformations

$$\hat{\delta}_{\hat{\lambda}} \hat{B}_{\mu\nu} = \hat{D}_{[\mu} \hat{\lambda}_{\nu]} , \qquad \hat{\delta}_{\hat{\lambda}} \hat{A}_\nu = 0 , \tag{3.7}$$

with gauge parameters $\hat{\lambda}_\mu$. The gauge algebra is abelian, $[\hat{\delta}_{\hat{\lambda}^1}, \hat{\delta}_{\hat{\lambda}^2}] = 0$ and reducible on-shell. Indeed, if $\hat{\lambda}_\mu = \hat{D}_\mu \hat{\eta}$, then the gauge transformation reduces to on-shell trivial transformations,

$$\hat{\delta}_{\hat{D}\hat{\eta}} \hat{B}_{\mu\nu} = \frac{1}{2}\left[\hat{F}_{\mu\nu} \stackrel{*}{,} \hat{\eta}\right] = \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\left[\frac{\delta \hat{I}}{\delta \hat{B}_{\rho\sigma}} \stackrel{*}{,} \hat{\eta}\right]. \tag{3.8}$$

3.3 Rigidity of gauge structure of commutative Freedman-Townsend model

If one sets $g = 0$, one gets the commutative Freedman-Townsend model,

$$I_0 = \int d^4x \ \mathrm{Tr}\ \left(-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} A_\mu A^\mu\right), \tag{3.9}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\,. \tag{3.10}$$

This model is invariant under the ordinary gauge transformations

$$\delta_\lambda B_{\mu\nu} = D_{[\mu}\lambda_{\nu]}\,, \qquad \delta_\lambda A_\nu = 0\,, \tag{3.11}$$

with on-shell gauge reducibility for $\lambda_\mu = D_\mu \eta$. The action (3.9) is the sum of a free, quadratic part plus a cubic interaction vertex proportional to

$$g_{AD} f^D{}_{BC} \epsilon^{\mu\nu\rho\sigma} B^A_{\mu\nu} A^B_\rho A^C_\sigma\,, \tag{3.12}$$

where $f^A{}_{BC}$ are the structure constants of $\mathrm{u}(N)$.

Obvious consistent deformations of the commutative Freedman-Townsend model are obtained by adding to (3.9) an arbitrary polynomial in the A_μ's and their derivatives.

Because these are strictly gauge invariant under (3.11) they do not modify the gauge structure. It turns out that these are the most general consistent deformations. Indeed, it has been shown in [19] that the Freedman-Townsend vertex (3.12), characterized by general structure constants $C^A{}_{BC}$ of a Lie-algebra, is the only gauge-symmetry deforming interaction vertices for a set of abelian 2-forms in 4 dimensions. That is, the most general deformation of (3.9) with $f^A{}_{BC} = 0$ is (3.9) with $f^A{}_{BC}$ replaced by the structure constants of an arbitrary Lie algebra, plus terms that involve only A_μ and its derivatives (see also [37, 38]). [There is no possibility for a Chern-Simons term $H \wedge B$, where H is the field strength of B, because $H \wedge B$ is a 5-form.] It easily follows from that result that the general first order deformation of the $\mathrm{u}(N)$-Freedman-Townsend model (3.9) is given again by the Freedman-Townsend action but with $f^A{}_{BC}$ replaced by $f^A{}_{BC} + gm^A{}_{BC}$, plus terms that involve only A_μ and its derivatives. The constants $m^A{}_{BC}$ are constrained by $m^A{}_{B[C} f^B{}_{DE]} + f^A{}_{B[C} m^B{}_{DE]}$ (Jacobi identity), i.e. define first-order deformations of the Lie algebra $\mathrm{u}(N)$. But this algebra is rigid (there is only one abelian factor) in the sense that one can bring $f^A{}_{BC} + gm^A{}_{BC}$ back to $f^A{}_{BC}$ by linear redefinitions in internal space. These are just particular canonical transformations (they would generate terms quadratic in A^A_μ if the metric is not invariant, but this is a deformation of the announced type). Thus, we may actually assume $m^A{}_{BC} = 0$, which means that up to redefinitions, the only deformations of the Freedman-Townsend model are precisely exhausted by the addition of functions of the A_μ and their derivatives. This implies, in particular, that the noncommutative Freedman-Townsend model must be amenable to the form

$$\hat{I} = I_0 + V[A; g] \,. \tag{3.13}$$

Note that the field A_μ is auxiliary in (3.9) — i.e. can be eliminated through its own equation of motion. It remains auxiliary in the deformed theory in the sense that its equations of motion can still be solved as formal power series in g. Note also that the equations of motion for $B_{\mu\nu}$ are left unchanged in the deformation and imply $F_{\mu\nu} = 0$. By solving this constraint, $A = g^{-1}dg$, the action becomes that of the non-linear sigma model modified by higher dimensionality operators.

4. Seiberg-Witten map for Freedman-Townsend model

4.1 Existence of the Seiberg-Witten map

The solution of the master equation for the non-commutative Freedman-Townsend model is given by

$$\hat{S} = \int d^4x \ \mathrm{Tr} \ \Big(-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \hat{B}_{\mu\nu} * \hat{F}_{\rho\sigma} + \frac{1}{2}\hat{A}_\mu * \hat{A}^\mu + \hat{B}^{*\mu\nu} * \hat{D}_{[\mu}\hat{C}_{\nu]} + \\ + \hat{C}^{*\mu} * \hat{D}_\mu \hat{\rho} + \frac{1}{8}\left[\hat{B}^{*\mu\nu} \overset{*}{,} \hat{B}^{*\rho\sigma}\right] \epsilon_{\mu\nu\rho\sigma} * \hat{\rho} \Big) \,. \tag{4.1}$$

while the solution of the master equation for the commutative case is given by the same expression with the $*$-product replaced by the ordinary product [39],

$$S_0 = \int d^4x \ \mathrm{Tr} \ \Big(-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \hat{B}_{\mu\nu} \hat{F}_{\rho\sigma} + \frac{1}{2}\hat{A}_\mu \hat{A}^\mu + \hat{B}^{*\mu\nu} \hat{D}_{[\mu}\hat{C}_{\nu]} + \\ + \hat{C}^{*\mu} \hat{D}_\mu \hat{\rho} + \frac{1}{8}\left[\hat{B}^{*\mu\nu}, \hat{B}^{*\rho\sigma}\right] \epsilon_{\mu\nu\rho\sigma} \hat{\rho} \Big) \,. \tag{4.2}$$

It follows from the cohomological analysis of the previous section that the Seiberg-Witten maps exists: by a canonical transformation, one can transform the

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functional (4.1) into the solution of the master equation for the commutative theory, plus $V_i([A_\mu])$-terms that are strictly gauge-invariant.[4]

One can explicitly construct the canonical transformation order by order in the fields, using standard cohomological weapons (homotopy formula for the free BRST differential etc.). We have not been able to sum the formal series obtained in this recursive manner in a concise and useful way, however, except in the u(1)-case, to which we shall therefore now exclusively turn.

4.2 SW map in the U(1) case

In the u(1)-case, the solution of the master equation for the commutative model is given by

$$S_0[\phi,\phi^*] = \int d^4x \left(-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}A_\mu A^\mu + B^{*\mu\nu}\partial_{[\mu}C_{\nu]} + C^{*\mu}\partial_\mu\rho\right), \qquad (4.3)$$

Our goal is to find a canonical transformation (2.12) such that (2.17) is satisfied for $\hat{S}[\hat{\phi},\hat{\phi}^*]$ given by (4.1) and $S_0[\phi,\phi^*]$ by (4.3). The searched-for generating functional $F[\phi,\hat{\phi}^*]$ takes the form

$$\begin{aligned} F\left[\phi,\hat{\phi}^*\right] = \int d^nx \;\; \Big(&\hat{A}^{*\mu} f_\mu([A],[H]) + \hat{B}^{*[\mu\nu]} f_{[\mu\nu]}([A],[B]) + \\ &+ \hat{C}^{*\mu}(C_\mu + 2[C_\alpha \stackrel{*}{,} f_\mu]^\alpha) - \\ &- \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\hat{B}^{*\mu\nu}\left[C_\alpha \stackrel{*}{,} \hat{B}^{*\rho\sigma}\right]^\alpha + \hat{\rho}^*\hat{\rho}\Big). \end{aligned} \qquad (4.4)$$

where the bracket $[\cdot \stackrel{*}{,} \cdot]^\alpha$ is defined by

$$[f \stackrel{*}{,} h]^\mu = -i\frac{g}{2}\theta^{\mu\nu} f *_1 \partial_\nu h\,, \qquad (4.5)$$

with

$$f *_1 g(x) = \frac{\sin \wedge_{12}}{\wedge_{12}} f(x_1)g(x_2)|_{x_1=x_2=x}\,, \qquad (4.6)$$

(see [40, 41] for further information on $*_1$). By construction, this bracket satisfies

$$[\partial_\mu f \stackrel{*}{,} h]^\mu = -\frac{1}{2}[f \stackrel{*}{,} h]\,. \qquad (4.7)$$

It is easy to check that in antifield number higher than 1, the generating functional (4.4) satisfies (2.17) for arbitrary $f_\mu([A],[H])$.

[4]Another way to arrive at the same conclusion, valid for Lie algebras that are non rigid, is the following. The only obstruction to the SW map arises if one "hits" the cubic vertex (3.12) in the non-commutative deformation process. But this cannot be the case, because of a direct power counting argument: the vertex (3.12) has dimension 5, while the first non-commutative vertex has dimension 6 — the field B has dimension 1 while the auxiliary field A has dimension 2.

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Identifying terms of antifield number 1 in (2.17) one gets,

$$f_{\mu\nu} = B_{\mu\nu} + 2[B_{\mu\alpha} \stackrel{*}{,} f_\nu]^\alpha - 2[B_{\nu\alpha} \stackrel{*}{,} f_\mu]^\alpha - \frac{ig\theta^{\alpha\beta}}{4} B_{\alpha\beta} *_1 \hat{F}^f_{\mu\nu} + [B_{\alpha\beta} \stackrel{*}{,} f_\mu \stackrel{*}{,} f_\nu]^{\alpha\beta}\,, \quad (4.8)$$

where

$$\hat{F}^f_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu + [f_\mu \stackrel{*}{,} f_\nu] \quad (4.9)$$

and

$$[B_{\alpha\beta} \stackrel{*}{,} f_\mu \stackrel{*}{,} f_\nu]^{\alpha\beta} = \sigma_0 \left([f_\mu \stackrel{*}{,} [C_\alpha \stackrel{*}{,} f_\nu]^\alpha] - [f_\nu \stackrel{*}{,} [C_\alpha \stackrel{*}{,} f_\mu]^\alpha] - [C_\alpha \stackrel{*}{,} [f_\mu \stackrel{*}{,} f_\nu]]^\alpha\right). \quad (4.10)$$

Here, σ_0 is a particular contracting homotopy for the differential γ_0 (longitudinal exterior derivative of the free theory) given in [42]). Explicitly, if instead of the variables $B_{\mu\nu}$, C_μ, ρ, and their derivatives, we define new variables through

$$\begin{aligned} y^\alpha &\equiv \partial_{(\nu_1} \dots \partial_{\nu_l} B_{[\mu)\lambda]}, \partial_{(\nu_1} \dots \partial_{\nu_l} C_{\mu)}\,, \\ z^\alpha &\equiv \partial_{(\nu_1} \dots \partial_{\nu_l} \partial_{[\mu)} C_{\lambda]}, -\partial_{(\nu_1} \dots \partial_{\nu_l} \partial_{\mu)} \hat{\rho}\,, \\ &\quad \rho, \partial_{(\nu_1} \dots \partial_{\nu_l} H_{\mu)\rho\sigma}\,, \end{aligned} \quad (4.11)$$

for $l = 0, 1, \dots$ and $H_{\mu\rho\sigma} = \partial_{[\mu} B_{\rho\sigma]}$, a particular expression for the contracting homotopy σ_0 is given by

$$\sigma_0 f = \int_0^1 \frac{dt}{t} \left[y^\alpha \frac{\partial f}{\partial z^\alpha}\right](ty, tz). \quad (4.12)$$

We leave it to the reader to check that the generating functional F does the job of transforming (4.1) into (4.3) no matter how the gauge-invariant functions $f_\mu([A],[H])$ is chosen (it must just be invertible). That there is an ambiguity in the SW map, characterized by addition of the gauge-invariant functions to f_μ, $f_{\mu\nu}$ and also to the higher antifield number terms in F (so that there are in fact *maps*) is not surprising: any redefinition that involves only gauge invariant quantities preserves the gauge structure.

To summarize, a particular class of solutions for the SW map for the non-commutative abelian Freedman-Townsend model has been obtained. The new feature of this map, compared with the SW map for Yang-Mills models, is that the generating functional $F[\phi, \hat{\phi}^*]$ is quadratic in some of the antifields, so that the transformations of some of the fields contain the antifields. This is related to the fact that the equations of motion appear in the reducibility identities (while the Yang-Mills gauge structure is defined not just on-shell, but also off-shell). This feature is easily incorporated at no cost in the antifield formalism.

Finally, we note that the lagrangian of the non-commutative Freedman-Townsend model can be mapped to the lagrangian of the commutative one up to second order in θ. It is an interesting question to investigate whether this holds to all orders. In this context, we recall that the 2-dimensional commutative and non-commutative WZW models are known to be equivalent not just in their gauge structure but also in their action [43, 44].

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5. Conclusion

The conclusion in [45] is "that there should be an underlying geometric reason for the Seiberg-Witten map." The analysis of this paper shows that there is at least a deformation theoretic reason for the existence of this map in the following sense.

Consistent deformations of gauge theories with non-trivial deformations of the gauge structure are in general severely constrained. The appropriate framework to study these constraints in the general case (reducibility, closure only on-shell) is the antifield-antibracket formalism. The cohomology that controls non-trivial deformations of the gauge structure is the local BRST cohomology, and how non-trivial cocycles depend on the antifields.

By analyzing explicitly the non-commutative Freedman-Townsend model, these considerations have been shown to apply beyond the original Yang-Mills framework. Because they do not depend on the precise deformation considered, they also apply for deformations that involve for instance more complicated star-products than the Weyl-Moyal star-product. The analysis can also be straightforwardly extended to models with higher rank p-forms as discussed in [37].

The cohomological theorems of [3, 4, 27], which guarantee the existence of the SW maps were studied initially with quantum motivations in mind (they control perturbative renormalizability — i.e. gauge invariance of the needed counterterms — as well as candidates anomalies for general (effective) theories with the same gauge structure as Yang-Mills models). The present paper clearly indicates their relevance in the classical context as well.

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